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STATING THE VARIATIONAL PROBLEMS OF GAS DYNAMICS

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STATING THE VARIATIONAL PROBLEMS OF GAS DYNAMICS *

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The gas dynamics equations of two-dimensional isoenergetic and isoentropic flows in the form of contour integrals have the form

$$\begin{aligned} &\oint f(y, w, \vartheta)\, dx - \varphi(y, w, \vartheta)\, dy = 0 \\ &\oint F(y, w, \vartheta)\, dx - \Phi(y, w, \vartheta)\, dy = 0 \end{aligned} \qquad \begin{pmatrix} w = w(x, y) \\ \vartheta = \vartheta(x, y) \end{pmatrix} \tag{1}$$

where x, y are the Descartes coordinates in the considered flow plane. w and ϑ are respectively the velocity module and the angle of inclination of the velocity toward the axis x. The integration is effected along an arbitrary closed contour L. The first of the equations (1) may be considered as the equation of motion in the projection on the axis x, and the second — as the equation of continuity.

In the differential form the equations have the form

$$\frac{\partial \varphi}{\partial x} + \frac{\partial f}{\partial y} = 0, \qquad \frac{\partial \Phi}{\partial x} + \frac{\partial F}{\partial y} = 0 \tag{2}$$

One of the possible ways of stating the problem of search for bodies with a minimum wave impedance (or nozzles with maximum thrust) amounts to the following:

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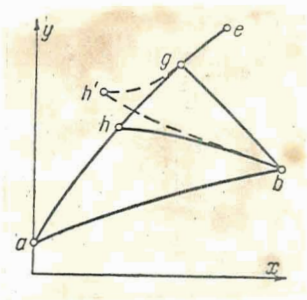
* K postanovke variatsionnykh zadach gazovoy dinamiki.

The points a and b and the characteristic of the incident flow ae are assugned (see fig.). A control contour ahb is introduced, with hb being the characteristic of equations (2), arriving at the point b. Equations (1) permit the writing the value χ of the resistance and the zero gas consumption Ψ through the generatrix ab in the form



$$\chi = \int_a^h f dx - \varphi dy + \int_h^b f dx - \varphi dy \qquad (3)$$

$$\Psi = 0 = \int_a^h F dx - \Phi dy + \int_h^b F dx - \Phi dy \qquad (4)$$

To resolve the problem it is necessary to find a function on hb, realizing the extremum of the functional (3) at conditions (4) and

$$X \equiv x_b - x_a = \int_a^h dx + \int_h^b dx \qquad (5)$$

taking into account the continuity of functions w and ϑ at the point h, and of the characteristic's equations, the first of which giving the relationship between dx and dy, and the other representing the so-called compatibility conditions.

Such variational problem setup takes into account all links on hb and it was utilized in the works [1 – 3].

Rao [4] proposed another approach to these problems' formulation. First an arbitrary line is selected as the closing line gb of the contour

agb, but the junction between w and ϑ on gb is not taken into account. This junction is conditioned by the following circumstances. Assume that the functions w and ϑ on gb are found by any method, say by the solution of the variational problem for example. Then the solution of the Cauchy problem for equations (2) with the data on gb defines the solution in the triangle gh'b. Generally speaking the characteristic gh' then found does not coincide with the portion gh of the assigned characteristic ae. An analogus picture takes place also in the case when the lines hb and gb exchange their roles. Therefore the statement of the problem in [4] is incomplete. The neglecting of one of the links leads in the general case to incorrect results. However, the complete statement of the problem is made difficult in the given case by the fact that the junction between w and ϑ on gb is unknown in an explicit form.

The above-presented discussions are evident, but the appearance of later works [5, 6], in which a great attention is given to the Rao method, and where no reference is made to the incorrectness of problem [4] statement, arouses the possibility of errors at examination of new problems. It must be pointed out that the basis of the Rao method in [5] concerns only the fulfillment of the condition of compatibility on the line gb found, with a characteristic direction. We will show that the final correct result have been found in [4] casually to a known extent. Equations (2) can be rewritten in the form

$$\begin{aligned} &\varphi_w w_x + \varphi_\vartheta \vartheta_x + f_w w_y + f_\vartheta \vartheta_y + f_y = 0 \\ &\Phi_w w_x + \Phi_\vartheta \vartheta_x + F_w w_y + F_\vartheta \vartheta_y + F_y = 0 \end{aligned} \qquad (6)$$

where the partial derivatives are marked by indices. The equations of system's (6) characteristics are determined by the equality

$$\begin{vmatrix} \varphi_w \tau - f_w & \varphi_\vartheta \tau - f_\vartheta \\ \Phi_w \tau - F_w & \Phi_\vartheta \tau - F_\vartheta \end{vmatrix} = 0 \qquad \left(\tau = \frac{dy}{dx}\right) \tag{7}$$

In its stating [4] the variational problem is reduced to searching for functions w and ϑ on gb, realizing the unconditional extremum of the functional

$$I = \int_a^g [\lambda_1 (f - \varphi\tau) + \lambda_2 (F - \Phi\tau) + \lambda_3]\, dx + \int_g^b [\lambda_1 (f - \varphi\tau) + \lambda_2 (F - \Phi\tau) + \lambda_3]\, dx \tag{8}$$

where $\lambda_1, \lambda_2, \lambda_3$ are Lagrange constant multipliers. The integral by ag is function of the upper limit. Effecting the variation we shall obtain in particular the Eiler equations on gb or on a portion of this line.

$$\lambda_1 (f_w - \varphi_w \tau) + \lambda_2 (F_w - \Phi_w \tau) = 0, \quad \lambda_1 (f_\vartheta - \varphi_\vartheta \tau) + \lambda_2 (F_\vartheta - \Phi_\vartheta \tau) = 0 \tag{9}$$

Generally speaking, the multipliers λ_1 and λ_2 are different from zero, and that is why the compatibility conditions of the equations (9) coincides with the equality (7) and gives unexpectedly the equation for the determination of the quantity $\tau = dy/dx$.

Let us now consider the variational problem in which the type (5) condition is substituted by a more general condition, not connected with the equations (1) :

$$P = \int_a^g U(y, w, \vartheta)\, dx - V(y, w, \vartheta)\, dy + \int_g^b U(y, w, \vartheta)\, dx - V(y, w, \vartheta)\, dy$$

In this case the type (8) functional is written in the form

$$J=\int_a^g [\lambda_1 (f-\varphi\tau)+\lambda_2 (F-\Phi\tau)+\lambda_3 (U-V\tau)]\,dx+$$
$$+\int_g^b [\lambda_1 (f-\varphi\tau)+\lambda_2 (F-\Phi\tau)+\lambda_3(U-V\tau)]dx$$

The Eiler equations on the line gb or on part of it have the form

$$\lambda_1 (f_w-\varphi_w\tau)+\lambda_2 (F_w-\Phi_w\tau)+\lambda_3 (U_w-V_w\tau)=0$$
$$\lambda_1 (f_\vartheta-\varphi_\vartheta\tau)+\lambda_2 (F_\vartheta-\Phi_\vartheta\tau)+\lambda_3 (U_\vartheta-V_\vartheta\tau)=0$$

and they do not determine τ. If we chose for gb a line that is not characteristic, the last equations will lead, as already noted, to results contracdicting the data on the characteristic ae.

Let us finally represent imaginarily, that when stating the initial problem, the junction between w and ϑ on gb is taken into account. In this case the expression under the integral along gb in the equality (8) will have an additional component. The corresponding Eiler equations will cease to be homogenous relative to λ_1 and λ_2. The complementary limitation on the type (7) - τ will not appear in this case, which points to the noncontra-dictoriness of the entire stating of the problem. The above-said is also valid for the case of nonisentropic flows.

***** THE END *****

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